TD PRIMITIVES et INTEGRALES

AMNS 22/23

Version 1

20 mars 2023

Exercice 99

a)

$$f(x) = x^2$$

$$\int f(x)\,dx = \int x^2\,dx$$

$$= \frac{x^3}{3} + C$$

b)

$$f(x) = \frac{1}{x}$$

$$\int f(x)\,dx = \ln|x| + C$$

c)

$$f(x) = x^{3/2}$$

$$\int f(x)\,dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2x^{\frac{5}{2}}}{5} + C$$

d)

$$f(x) = 4$$

$$\int f(x) = 4x + C$$

e)

$$f(x) = \cos(x)$$

$$\int f(x)\,dx = \sin(x) + C$$

f)

$$f(x) = 10^x$$

$$u^v = \exp(u \ln(v))$$

car $\ln(v^u) = u \ln(v)$

$$f(x) = 10^x$$

$$= \exp(\ln(10)\, x)$$

$$\int f(x)\,dx = \frac{e^{\ln(10)x}}{\ln(10)}$$

$$= \frac{10^x}{\ln(10)}$$

Exercice 100 :

a)

$$f(x) = 2x^2 + 3x$$

$$\int f(x)\,dx = \int (2x^2 + 3x)\,dx$$

$$= \int 2x^2\,dx + \int 3x\,dx$$

$$= 2\int x^2\,dx + 3\int x\,dx$$

$$= 2\frac{x^3}{3} + C_1 + \frac{3x^2}{2} + C_2$$

$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + C$$

b) $f(x) = \frac{\pi}{x}$

$$\int f(x)\,dx = \int \frac{\pi}{x}\,dx$$

$$= \pi \int \frac{1}{x}\,dx$$

$$= \pi \ln|x| + C$$

c)

$$f(x) = ax^{3/2} + bx^{-3/2}$$

a, b constantes.

$$\int f(x)\,dx = \int \left(ax^{3/2} + bx^{-3/2}\right) dx$$

$$= \int ax^{3/2}\,dx + \int bx^{-3/2}\,dx$$

$$= a\int x^{3/2}\,dx + b\int x^{-3/2}\,dx$$

$$= a\,\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C_1 + b\,\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C_2$$

$$= \frac{ax^{5/2}}{\frac{5}{2}} + C_1 + \frac{b\,x^{-\frac{1}{2}}}{-\frac{1}{2}} + C_2$$

$$= \frac{2a}{5}x^{\frac{5}{2}} + C_1 - 2b\,x^{-\frac{1}{2}} + C_2$$

$$= \frac{2a}{5} x^{\frac{5}{2}} - 2b x^{-\frac{1}{2}} + C$$

$$= \frac{2a}{5} x^{\frac{5}{2}} - \frac{2b}{\sqrt{x}} + C$$

avec $C = C_1 + C_2$

d) $f(x) = 4 + \ln(x)$

$$\int f(x)\,dx = \int (4 + \ln(x))\,dx$$

$$= \int 4\,dx + \int \ln(x)\,dx$$

$$= 4x + C_1 + x\ln(x) - x + C_2$$

$$= 3x + x\ln(x) + C_1 + C_2$$

$$= 3x + x\ln(x) + C$$

e)

$$f(x) = \alpha\cos(x) - \beta\tan(x) + \gamma\sin(x)$$

γ, β, γ constantes

$$\int f(x)\,dx = \int (\alpha\cos(x) - \beta\tan(x) + \gamma\sin(x))\,dx$$

$$= \int \alpha\cos(x)\,dx - \int \beta\tan(x)\,dx + \int \gamma\sin(x)\,dx$$

$$= \alpha\int \cos(x)\,dx - \beta\int \tan(x)\,dx + \gamma\int \sin(x)\,dx$$

$$= \alpha \sin(x) + C_1 + \beta \ln|\cos(x)| + C_2 - \gamma \cos(x) + C_3$$

$$= \alpha \sin(x) + \beta \ln|\cos(x)| - \gamma \cos(x) + C$$

Ici, on doit retenir que

$$\int \tan(x)\,dx = -\ln|\cos(x)| + C$$

f)

$$f(x) = 3 \cdot 10^x$$

$$= 3 \cdot \exp\left(x \ln(10)\right)$$

$$\int f(x)\,dx = \int 3 \exp\left(x \ln(10)\right) dx$$

$$= 3 \int \exp\left(x \ln(10)\right) dx$$

$$= \frac{3}{\ln(10)} \exp\left(x \ln(10)\right) + C$$

$$= \frac{3}{\ln(10)} 10^x + C$$

Exercice 101 :

a)

$$\int 5t^3 - 10t^{-6} + 4 \, dt = \int 5t^3 \, dt - \int 10t^{-6} \, dt + \int 4 \, dt$$

$$= 5\int t^3 \, dt - 10\int t^{-6} \, dt + 4\int dt$$

$$= 5\frac{t^4}{4} - 10\frac{t^{-5}}{-5} + 4t + C$$

$$= \frac{5t^4}{4} + 2t^{-5} + 4t + C$$

b)

$$\int (x^8 + x^{-8})\,dx = \int x^8\,dx + \int x^{-8}\,dx$$

$$= \frac{x^9}{9} + \frac{x^{-7}}{-7}$$

$$= \frac{x^9}{9} - \frac{x^{-7}}{7}$$

Remarque sur la mécanique

$$x^n \longrightarrow \frac{x^{n+1}}{n+1}$$

c)

$$\int \left(3\sqrt[4]{x^3} + \frac{7}{x^5} + \frac{1}{6\sqrt{x}}\right) dx$$

$$= 3\int \sqrt[4]{x^3}\, dx + 7\int x^{-5} dx + \frac{1}{6}\int x^{-\frac{1}{2}} dx$$

$$= 3\int x^{\frac{3}{4}} dx + 7\int x^{-5} dx + \frac{1}{6}\int x^{-\frac{1}{2}} dx$$

$$= \frac{4.3}{7} x^{\frac{7}{4}} - \frac{7}{4} x^{-4} + \frac{1}{3} x^{\frac{1}{2}} + C$$

$$= \frac{12}{7} x^{\frac{7}{4}} - \frac{7}{4} x^{-4} + \frac{x^{\frac{1}{2}}}{3} + C$$

d) $\int dy = \int 1\, dy$

$= y + C$

e) On developpe

$$(w+\sqrt[3]{w})(4-w^2) = 4w - w^3 + 4\sqrt[3]{w} - w^2 \cdot \sqrt[3]{w}$$

$$= 4w - w^3 + 4w^{\frac{1}{3}} - w^2 \cdot w^{\frac{1}{3}}$$

$$= 4w - w^3 + 4w^{\frac{1}{3}} - w^{\frac{6}{3}+\frac{1}{3}}$$

$$= 4w - w^3 + 4w^{1/3} - w^{7/3}$$

$$\int (w+\sqrt[3]{w})(4-w^2)\,dw = \int (4w - w^3 + 4w^{1/3} - w^{7/3})\,dw$$

$$= 4\int w\,dw - \int w^3\,dw + 4\int w^{1/3}\,dw - \int w^{7/3}\,dw$$

$$= \frac{4w^2}{2} - \frac{w^4}{4} + 3w^{4/3} - \frac{3}{10}w^{10/3}$$

$$= 2w^2 - \frac{w^4}{4} + 3w^{4/3} - \frac{3}{10}w^{10/3}$$

Exercice 102

a) $\int \frac{4x^{10} - 2x^4 + 15x^2}{x^3}\,dx = \int 4x^7 - 2x + 15x^{-1}\,dx$

$= 4\int x^7\,dx - 2\int x\,dx + 15\int \frac{1}{x}\,dx$

$= \frac{4x^8}{8} - \frac{2x^2}{2} + 15\ln(x) + C$

$= \frac{x^8}{2} - x^2 + 15\ln|x| + C$

b) $\int 3e^x + 5\cos(x)\,dx = 3\int e^x\,dx + 5\int \cos(x)\,dx$

$= 3e^x + 5\sin(x) + C$

c) $\int \frac{23}{\sqrt{1-y^2}}\,dy = 2\int \frac{1}{\sqrt{1-y^2}}\,dy = 2\arcsin(y) + C$

d) $\int \sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right)dt = \frac{1}{2}\int \sin(t)\,dt = -\frac{\cos(t)}{2} + C$

$\sin\left(\frac{t}{2} + \frac{t}{2}\right) = \sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right) + \cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{2}\right)$

Exercice 103 :

$$\int f(x)\,dx = \frac{2}{3}x^3 + \frac{3}{2}x^2 + C$$

$$\frac{d}{dx}\int f(x) = \frac{d}{dx}\left(\frac{2}{3}x^3 + \frac{3}{2}x^2 + C\right)$$

$$= 3\cdot\frac{2}{3}x^2 + 2\cdot\frac{3}{2}x = 2x^2 + 3x$$

$$= f(x)$$

Exercice 104 :

a) $\int e^{-ax}\,dx$

$$u = -ax$$

$$\frac{du}{dx} = -a$$

$$dx = -\frac{1}{a}\,du$$

$$\int e^{-ax}\,dx = -\frac{1}{a}\int e^{u}\,du$$

$$= -\frac{1}{a}e^{u} + C$$

$$= -\frac{1}{a}e^{-ax} + C$$

b) $\int \frac{1}{3x-2}\,dx$

$u = 3x-2$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3}du$

$\int \frac{1}{3x-2}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|3x-2| + C$

c) $\int (1-x)^{3/2}\,dx$

$u = 1-x$

$du = -dx$

$dx = -du$

c) $\int (1-x)^{3/2}\,dx$

$u = 1-x$

$du = -dx$

$dx = -du$

$$-\int u^{3/2}\,du = -\frac{2}{5}u^{\frac{5}{2}} + C$$

$$= -\frac{2}{5}(1-x)^{5/2} + C$$

d) $\int \frac{1}{\sqrt{2x+1}}\,dx$

$u = 2x+1$ $\quad du = 2dx$

$dx = \frac{1}{2}du$

$$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}\cdot\frac{1}{2}u^{\frac{1}{2}} + C$$

$$= \frac{1}{4}u^{\frac{1}{2}} + C$$

$$= \frac{1}{4}\sqrt{2x+1} + C$$

$$f(x) = h(g(x))\,g'(x)$$

$$\int f(x)\,dx = \int h(g(x))\,g'(x)\,dx$$

$$u = g(x)$$

$$\frac{du}{dx} = g'(x)$$

Exercice 105 :

a) $\int 3(8y-1)\,e^{4y^2-y}\,dy$

$$u = 4y^2 - y$$

$$\frac{du}{dy} = 8y - 1$$

$$du = (8y-1)\,dy$$

$$\int 3(8y-1)\,e^{4y^2-y}\,dy = 3\int e^u\,du = 3e^u + C$$

$$= 3e^{4y^2-y} + C$$

b) $\int x^2(3-10x^3)^4\,dx$

$u = 3-10x^3$ $\qquad u(x)$

$\frac{du}{dx} = -30x^2$

$x^2\,dx = -\frac{1}{30}\,du$

$$\int x^2(3-10x^3)^4\,dx = -\frac{1}{30}\int u^4\,du$$

$$= -\frac{1}{30}\cdot\frac{1}{5}u^5 + C$$

$$= -\frac{1}{150}(3-10x^3)^5 + C$$

c) $\int \frac{x}{\sqrt{1-4x^2}}\,dx$

$u = 1-4x^2$

$du = -8x\,dx$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx = -\frac{1}{8}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{2}{8}u^{\frac{1}{2}} + C$$

$$= -\frac{1}{4}(1-4x^2)^{1/2} + C$$

$$= \frac{-\sqrt{1-4x^2}}{4}$$

d)

$$\int \sin(1-x)(2-\cos(1-x))^4 dx$$

On pose $u = 2 - \cos(1-x)$

$$\frac{du}{dx} = -1 \cdot (+\sin(1-x))$$

où $(f \circ g)'(x) = f'(g(x))\, g'(x)$

$f(x) = -\cos(x)$

$g(x) = 1 - x$

$f'(x) = +\sin(x)$

$g'(x) = -1$

$$f'(g(x))\, g'(x) = +\sin(1-x) \cdot -1$$
$$= -\sin(1-x)$$

$$\frac{du}{dx} = -\sin(1-x)$$

$$du = -\sin(1-x)\,dx$$

$$\int \sin(1-x)(2-\cos(1-x))^4\,dx = -\int (2-\cos(1-x))^4(-\sin(1-x))\,dx$$

$$= -\int u^4\,du$$

$$= -\frac{u^5}{5} + C$$

$$= -\frac{(2-\cos(1-x))^5}{5} + C$$

e)

$$\int \cos(3z)\sin^{10}(3z)\,dz$$

$u = \sin(3z)$

$\frac{du}{dz} = 3\cos(3z)$

$du = 3\cos(3z)\,dz$

$\cos(3z)\,dz = \frac{1}{3}du$

$$\int \cos(3z)\sin^{10}(3z)\,dz = \int \sin^{10}(3z)\cos(3z)\,dz$$

$$= \frac{1}{3}\int u^{10}\,du$$

$$= \frac{1}{3}\cdot\frac{1}{11}u^{11} + C$$

Ainsi

$$\int \cos(3z)\sin^{10}(3z)\,dz = \frac{1}{33}\sin^{11}(3z)+C$$

f)

$$\int \frac{3}{5y+4}\,dy$$

$u = 5y+4$

$\frac{du}{dy} = 5$

$dy = \frac{1}{5}du$

$$\int \frac{3}{5y+4}\,dy = 3\int \frac{1}{5y+4}\,dy$$

$$= \frac{3}{5}\int \frac{1}{u}\,du$$

$$= \frac{3}{5}\ln|u| + C$$

$$= \frac{3}{5}\ln|5y+4| + C$$

g)

$$\int \frac{3y}{5y^2+4}\,dy$$

$$u = 5y^2 + 4$$

$$\frac{du}{dy} = 10y$$

$$y\,dy = \frac{1}{10}\,du$$

$$\int \frac{3y}{5y^2+4}\,dy = 3\int \frac{y}{5y^2+4}\,dy$$

$$= \frac{3}{10}\int \frac{1}{u}\,du$$

$$= \frac{3}{10}\ln|u| + C$$

$$= \frac{3}{10}\ln|5y^2+4| + C$$

h)

$$\int \frac{3y}{(5y^2+4)^2}\,dy$$

$u = 5y^2 + 4$

$\frac{du}{dy} = 10y$

$y\,dy = \frac{1}{10}\,du$

$$\int \frac{3y}{(5y^2+4)^2}\,dy = 3\int \frac{y}{(5y^2+4)^2}\,dy$$

$$= \frac{3}{10}\int \frac{1}{u^2}\,du$$

$$= \frac{3}{10}\int u^{-2}\,du$$

$$= \frac{3}{10}\cdot\frac{1}{-1}u^{-1} + C$$

$$= -\frac{3}{10}u^{-1} + C$$

$$= -\frac{3}{10}\cdot\frac{1}{5y^2+4} + C$$

i)

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

$$u = t^4 + 2t$$

$$\frac{du}{dt} = 4t^3 + 2 = 2(2t^3+1)$$

$$(2t^3+1)\,dt = \frac{1}{2}\,du$$

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \frac{1}{2}\int \frac{1}{u^3}\,du$$

$$= \frac{1}{2}\int u^{-3}\,du$$

$$= \frac{1}{2}\cdot\frac{1}{-2}\,u^{-2} + C$$

$$= -\frac{1}{4}\,u^{-2} + C$$

$$= -\frac{1}{4(t^4+2t)^2} + C$$

j) $\int \frac{2t^3+1}{t^4+2t}\,dt$

$u = t^4+2t$

$\frac{du}{dt} = 4t^3+2 = 2(2t^3+1)$

$\frac{1}{2}du = (2t^3+1)\,dt$

$\int \frac{2t^3+1}{t^4+2t}\,dt = \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|t^4+2t| + C$

k) $$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$u = 1-4x^2$

$\frac{du}{dx} = -8x$

$x\,dx = -\frac{1}{8}\,du$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx = -\frac{1}{8}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{1}{8}\cdot\frac{1}{-\frac{1}{2}+1}\,u^{-\frac{1}{2}+1} + C$$

$$= -\frac{1}{8}\cdot 2\,u^{\frac{1}{2}} + C$$

$$= -\frac{1}{4}u^{1/2} + C$$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx = -\frac{1}{4}\sqrt{1-4x^2} + C$$

Exercice 106

a)

$$\int_{-\pi}^{\pi} \sin(x)\,dx = \left[-\cos(x)\right]_{-\pi}^{\pi}$$

$$= -\cos(\pi) - \left(-\cos(-\pi)\right)$$

b)

Rappel : $\cos(-x) = \cos(x)$

$$\int_0^{+\infty} 4e^{-x}\,dx = 4\int_0^{+\infty} e^{-x}\,dx$$

$$= 4\left[-e^{-x}\right]_0^{+\infty}$$

$$= 4\left(\lim_{x\to+\infty}\left(-e^{-x}\right) - \left(-e^{-0}\right)\right)$$

$$= 4\lim_{x\to+\infty} -\frac{1}{e^x} + e^0$$

$$= 4.0 + 4.1$$
$$= 4$$

c)

$$\int_{-1}^{2} 12x^3 - 3x^2 + x - 1 \, dx = 12\int_{-1}^{2} x^3 \, dx - 3\int_{-1}^{2} x^2 dx + \int_{-1}^{2} x \, dx - \int_{-1}^{2} dx$$

$$= 12\left[\frac{x^4}{4}\right]_{-1}^{2} - 3\left[\frac{x^3}{3}\right]_{-1}^{2} + \left[\frac{x^2}{2}\right]_{-1}^{2} - \left[x\right]_{-1}^{2}$$

$$= \frac{12}{4}\left(2^4 - (-1)^4\right) - \frac{3}{3}\left(2^3 - (-1)^3\right) + \frac{1}{2}\left(2^2 - (-1)^2\right) - \left(2 - (-1)\right)$$

$$= 3(16 - 1) - (8 + 1) + \frac{1}{2}(2 + 1) - 3$$

$$= 45 - 9 + \frac{3}{2} - 3$$

$$= 33 + \frac{3}{2}$$

$$= \frac{66}{2} + \frac{3}{2}$$

$$= \frac{69}{2}$$

d)

$$\int_{w_1}^{w_2} t\sin(t) \, dt$$

$$\begin{cases} u = t \\ v' = \sin(t) \end{cases}$$

$$\begin{cases} u' = 1 \\ v = -\cos(t) \end{cases}$$

$$\int_{w_1}^{w_2} u\,v' = \left[uv\right]_{w_1}^{w_2} - \int_{w_1}^{w_2} u'v$$

$$\int_{w_1}^{w_2} t\sin(t)\,dt = \left[-t\cos(t)\right]_{w_1}^{w_2} + \int_{w_1}^{w_2} \cos(t)\,dt$$

$$= -w_2\cos(w_2) + w_1\cos(w_1) + \left[\sin(t)\right]_{w_1}^{w_2}$$

$$= -w_2\cos(w_2) + w_1\cos(w_1) + \sin(w_2) - \sin(w_1)$$

e)

$$\int_{-1}^{1} \frac{1}{x+2}\,dx = \left[\ln(|x+2|)\right]_{-1}^{1}$$

$$= \ln(3) - \ln(1)$$

$$= \ln(3)$$

f) $$\int_1^{\infty} \ln(x)\,dx = \left[x\ln(x) - x\right]_1^{\infty}$$

$$= \lim_{x \to \infty} \left(x\ln(x) - x\right) - \left(1\cdot\ln(1) - 1\right)$$

$$= \infty - (-1)$$

$$= \infty + 1$$

$$= \infty$$

Exercice 107)

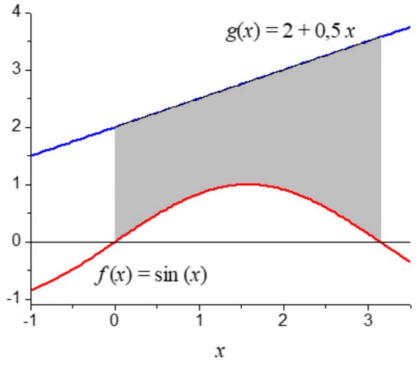


Figure 35.

L'aire en gris est égale à l'intégrale :

$$\int_0^{\pi} \left(2 + \frac{1}{2}x - \sin(x)\right) dx = 2\int_0^{\pi} dx + \frac{1}{2}\int_0^{\pi} x\,dx - \int_0^{\pi} \sin(x)\,dx$$

$$= 2\left[x\right]_0^{\pi} + \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\pi} - \left[-\cos(x)\right]_0^{\pi}$$

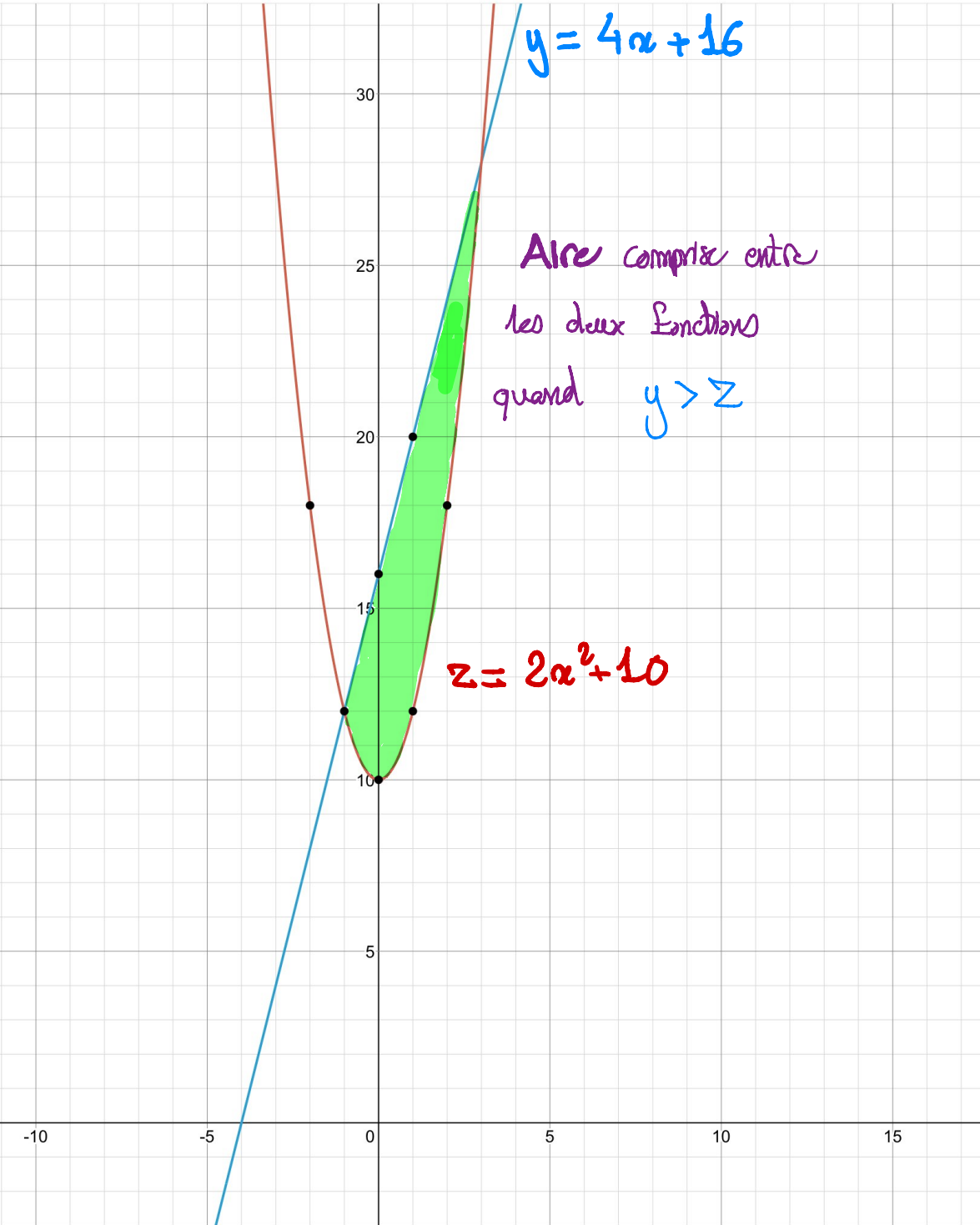
$$= 2(\pi - 0) + \frac{1}{2}\left(\frac{\pi^2}{2} - 0\right) - \left(-\cos(\pi) + \cos(0)\right)$$

$$= 2\pi + \frac{1}{4}\pi^2 + \cos(\pi) - \cos(0)$$

$$= 2\pi + \frac{\pi^2}{4} - 1 - 1$$

$$= 2\pi + \frac{\pi^2}{4} - 2$$

Exercice 108 :


y = 4x + 16
Aire comprise entre
les deux fonctions
quand y > z
z = 2x² + 10
30
25
20
15
10
5
-10
-5
0
5
10
15

$y = 4x + 16$

$z = 2x^2 + 10$

$y > z \iff 4x + 16 > 2x^2 + 10$

$\iff 4x + 16 - 2x^2 - 10 > 0$

$\iff -2x^2 + 4x + 6 > 0$

$\iff -x^2 + 2x + 3 > 0$

$\Delta = 2^2 - 4 \cdot (-1) \cdot 3$

$= 4 + 12 = 16$

$r_1 = \dfrac{-2 + \sqrt{16}}{-2} = \dfrac{2}{-2} = -1$

$r_2 = \dfrac{-2 - \sqrt{16}}{-2} = \dfrac{-6}{-2} = 3$

$-x^2 + 2x + 3 = -(x+1)(x-3)$

$= -(x^2 - 3x + x - 3)$

$= -x^2 + 2x + 3$ ✓

$\Leftrightarrow -x^2 + 2x + 3 > 0$

$\Leftrightarrow -(x+1)(x-3) > 0$

$\Leftrightarrow (x+1)(x-3) < 0$

$\Leftrightarrow$ $x > -1$ et $x < 3$ | $x < -1$ et $x > 3$

$ab < 0$

$\begin{cases} a < 0 \text{ et } b > 0 \\ a > 0 \text{ et } b < 0 \end{cases}$

x peut satisfaire

$x > -1$

et $x < 3$ ✓

mais x ne peut pas satisfaire

$x < -1$

et $x > 3$

$y - 2 = 4x + 16 - 2x^2 - 10$

$= -2x^2 + 4x + 6$

recherchée

Aire est égale à l'Intégrale

$$\int_{-1}^{3} (y-2)(x)\,dx = \int_{-1}^{3} -2x^2 + 4x + 6\,dx$$

$$= -2\int_{-1}^{3} x^2\,dx + 4\int_{-1}^{3} x\,dx + 6\int_{-1}^{3} dx$$

$$= -2\left[\frac{x^3}{3}\right]_{-1}^{3} + 4\left[\frac{x^2}{2}\right]_{-1}^{3} + 6\left[x\right]_{-1}^{3}$$

$$= -\frac{2}{3}\left(3^3 - (-1)^3\right) + 2\left(3^2 - (-1)^2\right) + 6(3+1)$$

$$= -\frac{2}{3}(27+1) + 2 \cdot 8 + 24$$

$$= -\frac{56}{3} + 16 + 24$$

$$= -\frac{56}{3} + 40 = -\frac{56}{3} + \frac{120}{3} = \frac{64}{3}$$

$$M = \frac{1}{b-a} \int_a^b f(x)\,dx$$

Exercice 109 :

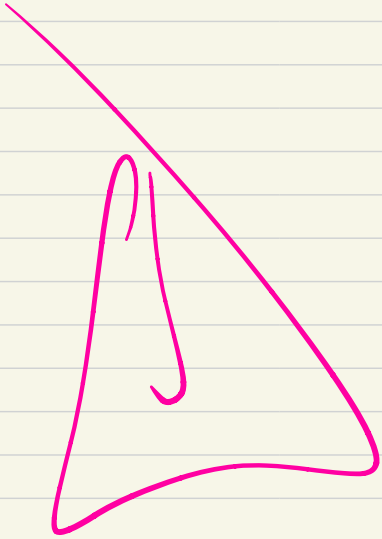
$$M_f = \frac{1}{1-0} \int_0^1 e^{-x}\,dx$$

$$= -\int_0^1 e^{x}\,dx$$

$$= \left[-e^{x}\right]_0^1$$

$$= -e^{1} + e^{0}$$

$$= -e + 1$$



Exercice 110 :

a) Valeur moyenne de $\cos(x)$ entre 0 et π :

$$\frac{1}{\pi - 0} \int_0^{\pi} \cos(x)\,dx = \frac{1}{\pi} \left[\sin(x)\right]_0^{\pi}$$

$$= \frac{1}{\pi} \left(\sin(\pi) - \sin(0)\right)$$

$$= \frac{1}{\pi} (-1)$$

$$= -\frac{1}{\pi}$$

b) Valeur moyenne de $\cos(x)$ entre 0 et $\frac{\pi}{2}$.

$$\frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} \cos(x)\,dx = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos(x)\,dx$$

$$= \frac{2}{\pi} \left[\sin(x)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{2}{\pi} \left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right)$$

$$= \frac{2}{\pi} (1 - 0)$$

$$= \frac{2}{\pi}$$

Exercício 111

$$v(t) = 12 - 3(t-2)^2$$
$$= 12 - 3(t^2 - 4t + 4)$$
$$= 12 - 3t^2 + 12t - 12$$
$$= -3t^2 + 12t$$

Velocidade média entre $t=0$ e $t=4$

$$\frac{1}{4-0}\int_0^4 v(t)\,dt = \frac{1}{4}\int_0^4 (-3t^2 + 12t)\,dt$$

$$= \frac{1}{4}\cdot(-3)\int_0^4 t^2\,dt + \frac{12}{4}\int_0^4 t\,dt$$

$$= -\frac{3}{4}\left[\frac{t^3}{3}\right]_0^4 + 3\left[\frac{t^2}{2}\right]_0^4$$

$$= -\frac{1}{4}(64) + \frac{3}{2}\cdot 16 = -16 + 24$$
$$= 8$$

Exercício 112 :

$$T = \int_a^b F\,dx$$

$F = -kx$

$$T = \int_a^b -kx\,dx$$

$$= -k \int_a^b x\,dx$$

$$= -k \left[\frac{x^2}{2}\right]_a^b$$

$$= -k \left(\frac{b^2}{2} - \frac{a^2}{2}\right)$$

$$= -\frac{k}{2}(b-a)(b+a)$$

$F = -\frac{k}{x^2}$

$$T = \int_a^b -\frac{k}{x^2}\,dx$$

$$= -k \int_a^b \frac{1}{x^2}\,dx$$

$$= -k \left[-\frac{1}{x}\right]_a^b$$

$$= -k \left(-\frac{1}{b} + \frac{1}{a}\right)$$

$$= -k \left(\frac{b-a}{ab}\right)$$

Exercice 113

$F = -Kx$ $\quad K = 0,1$ $\quad a = 1$, $b = 10$

$$T = \int_0^{10} -Kx\,dx$$

$$= \int_0^{10} -\frac{1}{10}x\,dx$$

$$= -\frac{1}{10}\int_0^{10} x\,dx = -\frac{1}{10}\left[\frac{x^2}{2}\right]_0^{10}$$

$$= -\frac{1}{10}\left(\frac{100}{2}\right) = -5$$

Exercice 114 :

$$I = \int_{-\infty}^{+\infty} \frac{b}{(\nu-\nu_0)^2 + b^2} \, d\nu$$

$$\int \frac{1}{1+u^2} \, du = \arctan(u) + C$$

$$I = \frac{\frac{1}{b^2}}{\frac{1}{b^2}} \quad I = \frac{b}{b^2} \int_{-\infty}^{\infty} \frac{1}{\frac{1}{b^2}\left((\nu-\nu_0)^2 + b^2\right)} \, d\nu$$

$$= \frac{1}{b} \int_{-\infty}^{\infty} \frac{1}{1 + \left(\frac{\nu-\nu_0}{b}\right)^2} \, d\nu$$

on pose $\quad u = \dfrac{\nu - \nu_0}{b}$

$$\frac{du}{d\nu} = \frac{1}{b}$$

$$d\nu = b \, du$$

$$I = \frac{b}{b} \int_{-\infty}^{\infty} \frac{1}{1+u^2} \, du = \left[\text{Arctan}(u)\right]_{-\infty}^{\infty}$$

$$\left[\operatorname{Arctan}(u)\right]_{-\infty}^{\infty} = \lim_{x\to\infty} \operatorname{Arctan}(x) - \lim_{x\to-\infty} \operatorname{Arctan}(x)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \pi$$

$$I = \pi$$

Exercice 115

a) $\int x\cos(x)\,dx$

$u = x$ | $u' = 1$

$v' = \cos(x)$ | $v = \sin(x)$

$$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$$

$$= x\sin(x) - (-\cos(x))$$

$$= x\sin(x) + \cos(x)$$

$$\int uv' = uv - \int u'v$$

b) $\int (3t+5)\cos\left(\frac{t}{4}\right) dt$

$u = 3t+5$	$u' = 3$
$v' = \cos\left(\frac{t}{4}\right)$	$v = 4\sin\left(\frac{t}{4}\right)$

$$\int \overset{u}{(3t+5)}\overset{v'}{\cos\left(\frac{t}{4}\right)} dt = \overset{u}{(3t+5)}\overset{v}{4\sin\left(\frac{t}{4}\right)} - \int 3\cdot 4\sin\left(\frac{t}{4}\right) dt$$

$$= 4(3t+5)\sin\left(\frac{t}{4}\right) - 12\int \sin\left(\frac{t}{4}\right) dt$$

$$= 4(3t+5)\sin\left(\frac{t}{4}\right) - 12\left(-4\cos\left(\frac{t}{4}\right)\right)$$

$$= 4(3t+5)\sin\left(\frac{t}{4}\right) + 48\cos\left(\frac{t}{4}\right)$$

c) $\int x\sqrt{x+1}\,dx$

$u = x$

$v' = \sqrt{x+1} = (x+1)^{\frac{1}{2}}$

$\int (x+1)^{\frac{1}{2}}\,dx$

$\phi = x+1$

$\frac{d\phi}{dx} = 1$

$\int (x+1)^{\frac{1}{2}}\,dx = \int \phi^{\frac{1}{2}}\,d\phi = \frac{\phi^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{2}{3}\phi^{\frac{3}{2}} + C$

$v' = (x+1)^{\frac{1}{2}}$

$v = \frac{2}{3}(x+1)^{\frac{3}{2}}$

$\int \underset{u}{x}\,\underset{v'}{\sqrt{x+1}}\,dx = \underset{uv}{\frac{2}{3}x(x+1)^{\frac{3}{2}}} - \frac{2}{3}\int \underset{u'v}{(x+1)^{\frac{3}{2}}}\,dx$

$= \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{2}{5}(x+1)^{\frac{5}{2}}$

$= \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{5/2}$ ✓

d) $\int \ln(x)\,dx = \int 1 \cdot \ln(x)\,dx$

$u' = 1$ $\quad u = x$

$v = \ln(x)$ $\quad v' = \frac{1}{x}$

$$\int u'v = uv - \int uv'$$

$$\int \ln(x) = x\ln(x) - \int x \cdot \frac{1}{x}\,dx$$

$$= x\ln(x) - \int dx$$

$$= x\ln(x) - x$$

Exercice 116

$$\frac{df(t)}{dt} = -at\,f(t)$$

$$\Leftrightarrow \quad \frac{df(t)}{f(t)} = -at\,dt$$

$$\Leftrightarrow \quad \int_{f(0)}^{f(x)} \frac{1}{f(t)}\,df(t) = -a\int_0^x t\,dt$$

$$\Leftrightarrow \quad \left[\ln(f(t))\right]_{f(0)}^{f(x)} = -a\left[\frac{t^2}{2}\right]_0^x$$

$$\Leftrightarrow \quad \ln(f(x)) - \ln(f(0)) = -\frac{a}{2}(x^2 - 0)$$

$$\Leftrightarrow \quad \ln(f(x)) - \ln(c) = -\frac{a}{2}x^2$$

$$\Leftrightarrow \quad \ln(f(x)) = \ln(c) - \frac{a}{2}x^2$$

$$\Leftrightarrow \quad f(x) = e^{\ln(c) - \frac{a}{2}x^2} = c\,e^{-\frac{a}{2}x^2}$$

Exercice 117

Résoudre

$$\frac{dG(x)}{dx} = \frac{a}{G(x)}$$

$$\Leftrightarrow \int_{G(0)}^{G(x)} G(y)\,\frac{dG(y)}{dy}\,dy = a\int_0^x dy$$

$G(0) = C$

$$\Leftrightarrow \frac{(G(x))^2}{2} - \frac{C^2}{2} = ax$$

$$\Leftrightarrow (G(x))^2 = 2ax + C^2$$

$$\Leftrightarrow G(x) = \pm\sqrt{2ax + C^2}$$

on vérifie :

$$\frac{dG(x)}{dx} = 2a \cdot \frac{1}{2\sqrt{2ax + C^2}}$$

$$= \frac{a}{\sqrt{2ax + C^2}} = \frac{a}{G(x)}$$

Exercice 118

$$F'(t) = \frac{1}{F^2(t)}$$

$$\frac{dF(t)}{dt} = \frac{1}{F^2(t)}$$

$$\int_{F(0)}^{F(t)} F^2(y) \frac{dF(y)}{dy} dy = \int_0^t dy$$

$$\Leftrightarrow \frac{1}{3}\left(F^3(t) - F^3(0)\right) = t$$

$$\Leftrightarrow F^3(t) = 3t + 1000$$

$$\Leftrightarrow F^3(t) = 3\left(t + \frac{1000}{3}\right)$$

$$\Leftrightarrow \begin{cases} F(t) = \sqrt[3]{3}\sqrt[3]{t + \frac{1000}{3}} \\ F(t) = (-1)^{2/3}\sqrt[3]{3}\sqrt[3]{t + \frac{1000}{3}} \\ F(t) = -\sqrt[3]{-3}\sqrt[3]{t + \frac{1000}{3}} \end{cases}$$

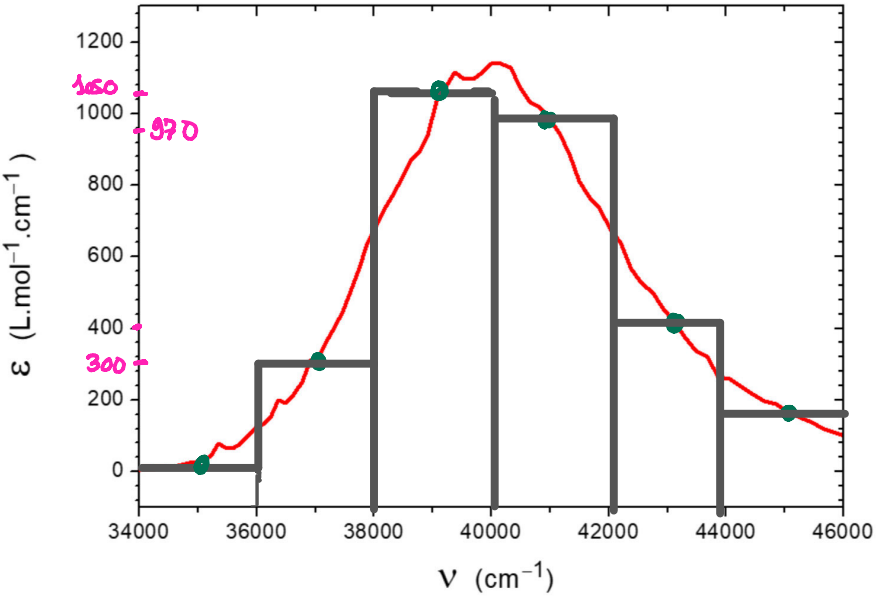

1200
1000
800
600
400
200
0
1150
970
300
ε (L.mol⁻¹.cm⁻¹)
34000
36000
38000
40000
42000
44000
46000
ν (cm⁻¹)

Figure 38.

6 morceaux de largeur 2000.

$f = 0 \cdot 2000 + 300 \cdot 2000 + 1050 \cdot 2000 + 970 \cdot 2000 + 400 \cdot 2000 + 1752000$

$= 5790000$